Quintessential Enhancement of Dark Matter Abundance

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Abstract

The presence of a dynamical scalar field in the early universe could significantly affect the 'freeze-out' time of particle species. In particular, it was recently shown that an enhancement of the relic abundance of neutralinos can be produced in this way. We examine under which conditions this primordial scalar field could be identified with the Quintessence scalar and find, through concrete examples, that modifications to the standard paradigm are necessary. We discuss two possible cases: the presence of more scalars and the switching on of an interaction.

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1 Introduction

As it is well known, according to the standard paradigm [1] a particle species goes through two main regimes during the cosmological evolution. At early times each constituent of the universe is in thermal equilibrium, a condition which is maintained until the particle interaction rate Γ remains larger than the expansion rate H. At some time, however, H will overcome Γ because the particles will be so diluted by the expansion that they will not interact anymore. The epoch at which $\Gamma = H$ is called 'freeze-out', and after that time the number of particles per comoving volume for that given species will stay constant for the remaining cosmological history. This is how cold dark matter particle relics (neutralinos, for example) are generated. This framework, although simple in principle, can be very complicated in practice. On one hand, we need a particle theory in order to compute Γ, on the other we have to choose a cosmological model to specify H. Even a small change in Γ and/or H would result in a delay or anticipation of the 'freeze-out' time of a given particle species and, as a consequence, in a measurable change in the relic abundance observed today.

The standard scenario [1] assumes that before Big Bang Nucleosynthesis (BBN), the dominant cosmological component was radiation and so the Hubble parameter was evolving according to $H^2 \sim \rho_r \sim a^{-4}$, where ρ_r is the energy density of radiation and a is the scale factor of the universe. This is a reasonable assumption, but the available data do not exclude modifications to this scenario. For example, it is conceivable that in the pre-BBN era, a scalar field had dominated the expansion for some time[1], leaving room to radiation only afterwards. To be more concrete, if we imagine to add a significant fraction of scalar energy density to the background radiation at some time in the past, this would produce a variation in H^2, depending on the scalar equation of state w_ϕ.[2] If $w_\phi > w_r = 1/3$, the scalar energy density would decay more rapidly than radiation, but temporarily increase the global expansion rate. This possibility was

[1] We mean, of course, after the end of inflation.

[2] Remember that the energy density of each cosmological component scales as $\rho_x/\rho_x^o = (a/a_o)^{-3(w_x+1)}$, if w_x is the corresponding equation of state.

explicitly considered in Ref. [2], where it was calculated that a huge enhancement of the relic abundance of neutralinos could be produced in this way.

The effect of an early scalar field dominance on electroweak baryogenesis is discussed in Ref.[4]. An alternative possibility for non-standard 'freeze-out' is proposed in Ref.[5].

In this paper we will recall how a period of scalar 'kination' (see below) could affect the relic density of neutralinos and discuss if this primordial scalar field could be identified with the Quintessence scalar, *i.e.* the field thought to be responsible for the present acceleration of the universe [3]. We will find that modifications to the standard Quintessence paradigm are necessary and discuss some concrete examples.

2 Scalar field 'kination'

The early evolution of a cosmological scalar field ϕ with a runaway potential $V(\phi)$ is typically characterised by a period of so–called 'kination' [6, 7, 8], during which the scalar energy density

$$\rho_\phi \equiv \frac{\dot{\phi}^2}{2} + V(\phi) \tag{1}$$

is dominated by the kinetic contribution $E_k = \dot{\phi}^2/2 \gg V(\phi)$. After this initial phase, the field comes to a stop and remains nearly constant for some time ('freezing' phase), until it eventually reaches an attractor solution[3]. A simple and interesting example is that of inverse power law potentials:

$$V(\phi) = M^{4+n}\ \phi^{-n} \tag{2}$$

with M a mass scale and n a positive number. These potentials exhibit the attractive feature of a stable attractor solution characterised by a constant scalar equation of state [6, 9]

$$w_n = \frac{nw - 2}{n + 2} \tag{3}$$

which depends only on the exponent n in the potential and on the background equation of state w. Since n is positive, the condition $w_n < w$ always holds and the scalar field, which can be sub-dominant at the beginning, will eventually overtake the background energy density. This is a welcome feature if we are modelling the present acceleration of the universe through the scalar field dynamics (Quintessence), since during matter domination ($w = w_m = 0$) the attractor has negative equation of state for any n.

In general, a scalar field in a cosmological setting obeys the evolution equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{4}$$

and, for any given time during the cosmological evolution, the relative importance of the scalar energy density w.r.t. to matter and radiation in the total energy density ρ

$$\rho \equiv \rho_m + \rho_r + \rho_\phi \tag{5}$$

depends on the initial conditions, and is constrained by the available cosmological data on the expansion rate and large scale structure. As it is well known, the cosmological expansion rate is governed by the Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_p^2}\ \rho \tag{6}$$

[3]For a detailed discussion of the existence and stability of attractor solutions for general potentials see [7, 8].

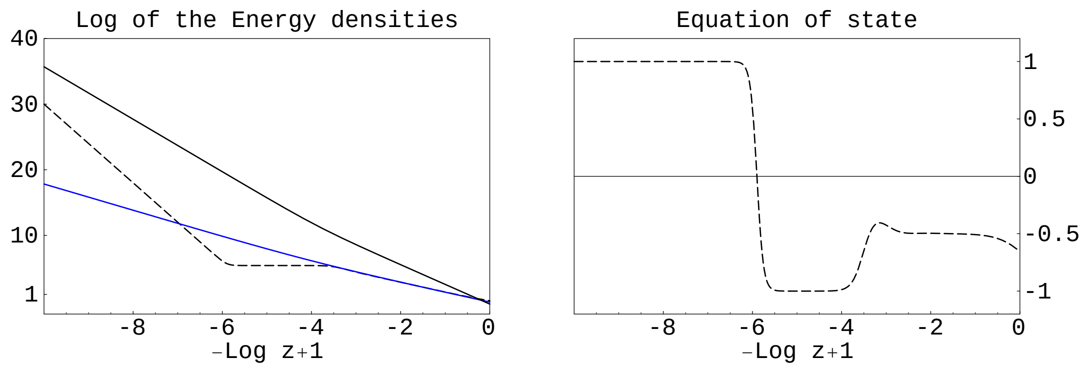

Figure 1: The figures show the typical evolution of the relevant energy densities (w.r.t. the present critical energy density $\rho_c^o \simeq 10^{-47} GeV^4$) and of the scalar equation of state, for a cosmological scalar field with potential $V \sim \phi^{-2}$. The black curve corresponds to the background (radiation plus matter) and the blue curve to the attractor. The dashed lines show the scalar energy density and equation of state: it can be easily seen that after an initial stage of 'kination' ($w_\phi = 1$), the field is 'freezing' ($w_\phi = -1$) and subsequently joins the attractor until it overtakes the background energy density. On the attractor the scalar equation of state is $w_\phi = -1/2$.

where ρ includes all the contributions of eq.(5), and we have assumed a spatially flat universe. Then, if we modify the standard picture according to which only radiation plays a role in the post-inflationary era and suppose that at some time $\hat{t}$ the scalar contribution was small but non negligible w.r.t. radiation, then at that time the expansion rate $H(\hat{t})$ should be correspondingly modified[4]. Since during the kination phase the scalar to radiation energy density ratio evolves like $\rho_\phi/\rho_r \sim a^{-3(w_\phi - w_r)} = a^{-2}$, the scalar contribution would rapidly fall off and leave room to radiation domination.

Is this of any interest to us? The answer is yes, because there is a clear cosmological signature of this early phase: the relic density of neutralinos [2]. The reasoning goes as follows: since the fall off of ρ_ϕ is so rapid during kination, we can respect the BBN bounds and at the same time keep a significant scalar contribution to the total energy density just few red-shifts before. For example, a scalar to radiation ratio $\rho_\phi/\rho_r = 0.01$ at BBN ($z \simeq 10^9$) would imply $\rho_\phi/\rho_r = 0.1$ at $z \simeq 3.16 \times 10^9$ and $\rho_\phi/\rho_r = 1$ at $z \simeq 10^{10}$, if the scalar field is undergoing a kination phase. As extensively discussed in [2], calculations of the relic densities of dark matter (DM) particles are usually done under the assumption that the universe is dominated by radiation while they decouple from the primordial plasma and reach their final relic abundance. However, as we have seen, it is conceivable that the scalar energy density respects the BBN bounds and at the same time contributes significantly to the total energy density at the time DM particles decouple. Indeed, an increase in the expansion rate H due to the additional scalar contribution would anticipate the decoupling of particle species and result in a net increase of the corresponding relic densities. As shown in [2], a scalar to radiation energy density ratio $\rho_\phi/\rho_r \simeq 0.01$ at BBN would give an enhancement of the neutralino codensity of roughly three orders of magnitude.

[4]We recall that the available data do not exclude that the scalar contribution could have been important for some time in the pre-BBN era. However, and this is our point, future observations might be able to explore the consequences of this possibility.

3 Quintessence?

As discussed in the previous section, the enhancement of the relic density of neutralinos requires that at some early time the scalar energy density was dominating the Universe. This fact raises a problem if we want to identify the scalar contribution responsible for this phenomenon with the Quintessence field [3] which (we suppose) accelerates the Universe today. Indeed, the scalar initial conditions are crucial to establish the scalar energy density contribution at any time.

In particular, the range of initial conditions which give rise to a non-negligible Quintessence contribution at present is huge but nonetheless does not include the case of a dominating scalar field at the beginning. In other words, the initial conditions must be such that the scalar energy density is sub-dominant (or, at most, of the same order of magnitude of ρ_r) at the beginning, if we want the Quintessence field to reach the cosmological attractor in time to be responsible for the presently observed acceleration of the expansion [6]. For initial conditions $\rho_\phi \gtrsim \rho_r$ we obtain the so-called 'overshooting' behaviour: the scalar field rapidly rolls down the potential and after the kination stage remains frozen at an energy density which is much smaller than the critical one. The larger is the ratio ρ_ϕ/ρ_r at the beginning, the smaller will be the ratio ρ_ϕ/ρ_c^o today.

There is also another situation in which the attractor is not reached in time. If the initial conditions are such that $\rho_\phi \lesssim \rho_c^o$ (the initial scalar density is smaller than the present critical energy density), then the scalar field would remain frozen throughout the whole cosmological history and join the attractor only beyond the present time. In this case the ratio ρ_ϕ/ρ_c^o remains unchanged and smaller than one until today (this the so-called 'undershooting' behaviour).

We should notice, however, that these rules strictly apply only to the standard case of a single uncoupled field with an inverse power law potential $V \sim \phi^{-n}$. As shown in [10] more complicated dynamics are possible if we relax this hypothesis and consider more general situations. The presence of several scalars and/or of a small coupling with the dark matter fields could modify the dynamics in such a way that the attractor is reached in time even if we started, for example, in the overshooting region.

More fields. Consider a potential of the form

$$V(\phi_1, \phi_2) = M^{n+4} \, (\phi_1 \phi_2)^{-n/2} \tag{7}$$

with M a constant of dimension mass. In this case, as discussed first in [10], the two fields' dynamics enlarges the range of possible initial conditions for obtaining a quintessential behaviour today. This is due to the fact that the presence of more fields allows to play with the initial conditions in the fields' values, while maintaining the total initial scalar energy density fixed. Doing so, it is possible to obtain a situation in which for a fixed ρ_ϕ^{in} in the overshooting region, if we keep initially $\phi_1 = \phi_2$ we actually produce an overshooting behaviour, while if we choose to start with $\phi_1 \neq \phi_2$ (and *the same* ρ_ϕ^{in}) it is possible to reach the attractor in time. This different behaviour emerges from the fact that, if at the beginning $\phi_2 \ll \phi_1$ then, in the example at hand, $\partial V/\partial\phi_2 \gg \partial V/\partial\phi_1$ and so ϕ_2 (the smaller field) will run away more rapidly and tend to overshoot the attractor, while ϕ_1 (the larger field) will move more slowly, join the attractor trajectory well before the present epoch and drive the total scalar energy density towards the required value[5]. In figure 2 the comparison between different cosmological evolutions depending

[5] We should specify that this interplay between the two fields is successful only until they both remain smaller than 1 in Planck units. Otherwise, also ϕ_1 would exit the allowed region for reaching the attractor in time and the behaviour of the total energy density would be worsened w.r.t. the equal fields' case. This fact can be checked numerically but has also a qualitative explanation. The key of the two-fields mechanism is that one field starts its cosmological evolution far from the attractor region, while the other is kept sufficiently close in order to join it in time. This does not work anymore if both fields start too far away from the attractor (even though on opposite sides). In fact, if we allow ϕ_1 to become larger than 1 initially, then this would correspond to push it down to the 'undershooting' region and prevent it as well to reach the attractor in time.

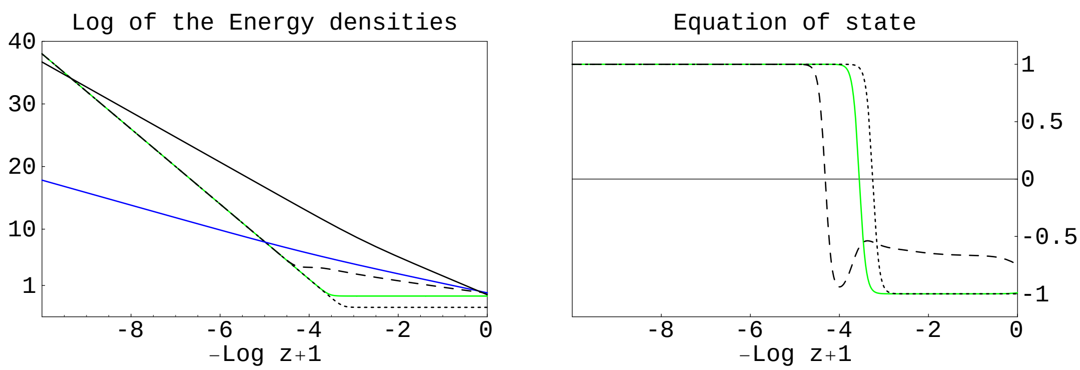

Figure 2: The figures show the evolution of the relevant energy densities (w.r.t. the present critical energy density $\rho_c^o \simeq 10^{-47} GeV^4$) and of the scalar equation of state, depending on the initial conditions, in the case of two scalar fields ϕ_1 and ϕ_2 with potential $V \sim (\phi_1 \phi_2)^{-1}$. The black curve corresponds to the background (radiation plus matter) and the blue curve to the attractor. The green line is the case in which the two fields start with the same initial conditions with a total energy density corresponding to the overshooting case. If we vary the fields' values at the beginning (keeping the total energy density fixed), we can obtain two situations: if both the fields are still $\ll 1$ at the beginning (dashed line) then the attractor is reached in advance w.r.t the equal fields' case; if instead one of the fields is > 1 (dotted line), then the attractor is reached later. In the examples shown, at $z = 10^{10}$ we have $\rho_\phi = 10^{38}$ and for the fields: $\phi_1 = \phi_2 = 10^{-19}$ (green); $\phi_1 = 10^{-16}$, $\phi_2 = 10^{-22}$ (dashed) and $\phi_1 = 100$, $\phi_2 = 10^{-40}$ (dotted).

on the fields' initial conditions, keeping ρ_ϕ^{in} fixed, is illustrated.

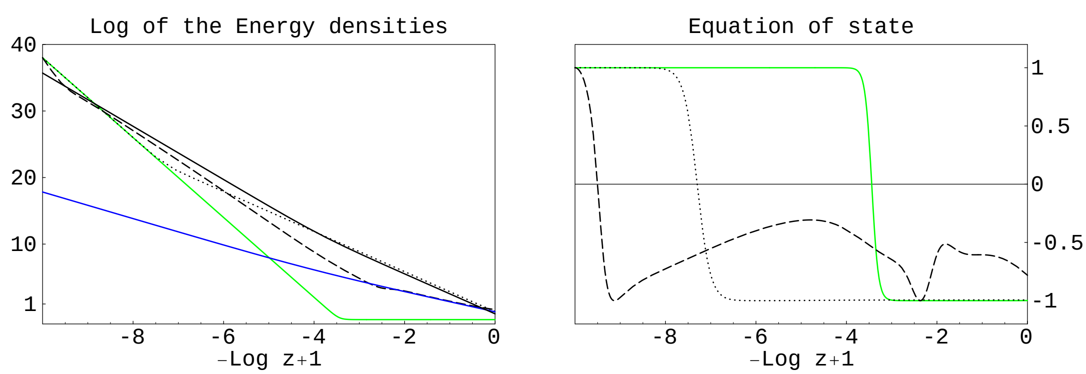

Figure 3: The figures show the evolution of the relevant energy densities (w.r.t. the present critical energy density $\rho_c^o \simeq 10^{-47} GeV^4$) and of the scalar equation of state, depending on the interaction of the Quintessence field with the dark matter fields. The black curve corresponds to the background (radiation plus matter) and the blue curve to the attractor. The green line is the Quintessence field evolution in the overshooting case ($\rho_\phi = 10^{38}$ at $z = 10^{10}$), with potential $V \sim \phi^{-2}$ and no interaction. If we switch on an additional term in the potential, the cosmological evolution will change correspondingly. The dashed line shows the case of a coupling $V_b = \frac{1}{2} b H^2 \phi^2$ with $b = 0.25$; the dotted line shows the case of a coupling $V_c = c \rho_m \phi$ with $c = 0.5$. Please note that the values chosen for b and c in the figure are purely illustrative. Coupling constants two orders of magnitude smaller than the ones considered here are sufficient to ensure the desired effect.

Interaction. Suppose now that the Quintessence scalar is not completely decoupled from the rest of the Universe. Among the possible interactions, as will be discussed below, two interesting cases are the following:

$$V_b = \frac{b}{2}\, H^2\phi^2 \qquad \text{or} \qquad V_c = c\rho_m\phi \tag{8}$$

If we add V_b or V_c to the potential $V = M^{n+4}\phi^{-n}$, the cosmological evolution will be accordingly modified. The main effect is that now the potential acquires a (time-dependent) minimum and so the scalar field is prevented from running freely to infinity. As a result, the long freezing phase that characterises the evolution of a scalar field with initial conditions in the overshooting region can be avoided. As can be seen in figure 3, the interactions in eq. (8) drive the scalar field trajectory towards the attractor (in the case of V_b) or towards ρ_m (in the case of V_c) well before the non-interacting case.

Effective interaction terms like V_b in eq. (8) were first introduced in Ref. [11] and are more recently discussed in Ref. [12]. The point is that supersymmetry breaking effects in the early universe can induce mass corrections to the scalar Lagrangian of order H^2. Indeed, if a term like $\delta K \sim \chi^*\chi\phi^*\phi$ is present in the Kahler potential, where χ is a field whose energy density is dominating the universe, this will result in a correction proportional to $\rho_\chi\phi^*\phi$ in the Lagrangian. Then, if the universe is critical $\rho_\chi \sim H^2$ and we obtain a mass correction for ϕ which goes like $\delta V \sim H^2\phi^2$.

The second type of interaction (V_c in eq. (8)) emerges in the context of scalar-tensor theories of gravity, in which a metric coupling exists between matter fields and massless scalars[6]. These theories, expressed in the so-called 'Einstein frame' are defined by the action (see, for example, [14]):

$$S = S_g + S_m \tag{9}$$

where $S_m = S_m[\Psi_m, A^2(\phi)g_{\mu\nu}]$ is the matter action which includes the scalar interaction via the multiplicative factor $A^2(\phi)$ before the metric tensor $g_{\mu\nu}$, and the gravitational action reads

$$S_g = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R - 2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi)] \quad . \tag{10}$$

The scalar field equation in this context is modified w.r.t. eq.(4) by the presence of an additional source term

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{2}\frac{dV}{d\phi} = -4\pi G\alpha(\phi)T \tag{11}$$

where $\alpha(\phi) \equiv d\log A(\phi)/d\phi$ and T is the trace of the matter energy-momentum tensor $T^{\mu\nu}$. The case $\alpha(\phi) = 0$ (*i.e.* $A(\phi) =$ const.) corresponds to the standard scenario with the scalar field decoupled from matter fields; while it can be easily seen that eq.(11) is equivalent to switching on an interaction term like V_c of eq. (8), if we choose the function $A(\phi)$ to be an exponential.

As it is well known, introducing an interaction between the matter fields and a light scalar should always done with great care in order to avoid unwanted effects like time variation of constants and modification of gravitational laws (for discussions of these issues in the Quintessence context see Refs.[10, 15, 16]). Limits on the possible values of the couplings b and c in eq. (8) depend on the details of the theory that originates them and on the cosmological epoch we are considering. Just as a rough estimate, we recall that at the present time solar system measurements impose on metric theories of gravity (see [14]) an upper bound for c of order 10^{-1}.

4 Conclusions

In this letter we have shown that modifications to the standard Quintessence paradigm are possible in order to make the Quintessence scalar responsible of an enhancement of the relic

[6]For a detailed discussion of these theories in the context of Quintessence cosmology see, for example, Refs.[13].

density of neutralinos. We have illustrated through specific examples that this can be obtained in two different ways: by considering more scalar fields in the Quintessence fluid and introducing an interaction term in the scalar potential.

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